

CTF Crypto

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Feeling of Randomness

- Plaintext is always something we might know about, some pattern. i.e - for two messages m_1, m_2 - $P(m_1) \neq P(m_2)$. For example, if we toss a coin - $P(\text{"heads"}) = \frac{1}{2}$, $P(\text{"tails"}) = \frac{1}{2}$ $P(\text{"hello world"}) = 0$
- But Ciphertext is generally random.
- But this probabilistic disparity in plaintext sometimes leads to breaking of complete cipher.

Must have tools

1. Python - both 2 and 3

- a. 2 is used when you know you have supported software and need simplicity
- b. 3 when need to integrate shit code from internet
- c. Pro tip: Always refer internet when converting *bytearray*, *bytes* and *string*.
- d. Install pwntools in both
- e. Install pycrypto

2. Sage Math

- a. Optional, never used personally
- b. Scar uses and solving group theory and ecc things is very easy using sagemath

3. A habit of backing up multiple times

- a. Things take time to generate in crypto, good to print everything out or even better write to file as well.

Terminology

1. Symmetric Encryption

$$E(m, k) = c \quad D(c, k) = m$$

2. Asymmetric Encryption

$$\text{Keygen}() = \text{pub, priv} \quad E(m, \text{pub}) = c \quad D(c, \text{priv}) = m$$

3. Cryptographic Hashing

$$H(m) = h \quad h \rightarrow m \text{ not possible}$$

4. Signing

$$\text{Keygen}() = \text{pub, priv} \quad \text{Sign}(m, \text{priv}) = s \quad \text{Verify}(m, s, \text{pub}) = \text{t/f}$$

pwntools

- Interacting with programs hosted with nc (nc is just like running terminal programs, running on remote computer)

```
>>> conn = remote('ftp.ubuntu.com',21)
>>> conn.recvline() # doctest: +ELLIPSIS
b'220 ...'
>>> conn.send(b'USER anonymous\r\n')
>>> conn.recvuntil(b' ', drop=True)
b'331'
>>> conn.recvline()
b'Please specify the password.\r\n'
>>> conn.close()
```

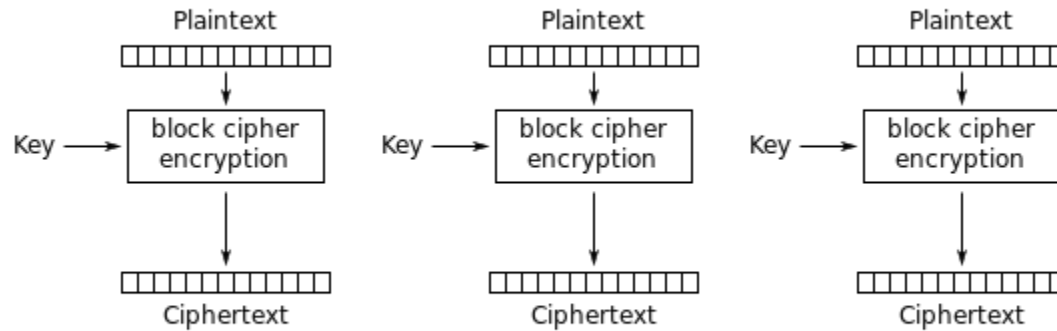
Classical Ciphers

- Substitution Cipher
 - divide the data into blocks, run a deterministic function on it, concatenate all result blocks
 - Attack - Frequency analysis attack
- Single byte Xor Cipher
 - $ct[i] = p[i] \oplus k$
 - Attack - Frequency analysis attack
- Multibyte Xor Cipher
 - $k = m\text{-byte key}, ct[i] = p[i] \oplus k[i \% m]$
 - Take 1st, $(m + 1)$ th, $(2m+1)$ th.... Byte, concatenate, run single byte xor cipher breaker on it
 - Xortool

AES

- AES is a block cipher. It converts 128 bit (16 bytes) of plaintext into 128 bit or ciphertext using a key. It has three variants depending on key size. Bigger keysize means better security (debatable). AES-128, AES-196 and AES-256
- No need to know what's inside it. For CTF purposes, assume that without key, you cannot encrypt/decrypt.
- As it only encrypts 128 bits, we developed some tricks to encrypt arbitrary length plaintexts called block modes.
- Note: Block modes are not limited to AES, but for any block cipher.

ECB Mode



Electronic Codebook (ECB) mode encryption

ECB Mode Security Problem - Non Diffusion

Assume block size is 8 bit

ECB("A") = "X"

ECB("B") = "Y"

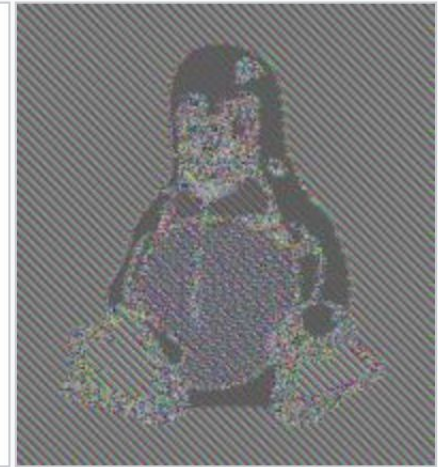
ECB("ABBA") = "XYYX"

Simple Substitution Cipher if the key is fixed.

Assignment: Make the penguin on the right using PIL and AES.

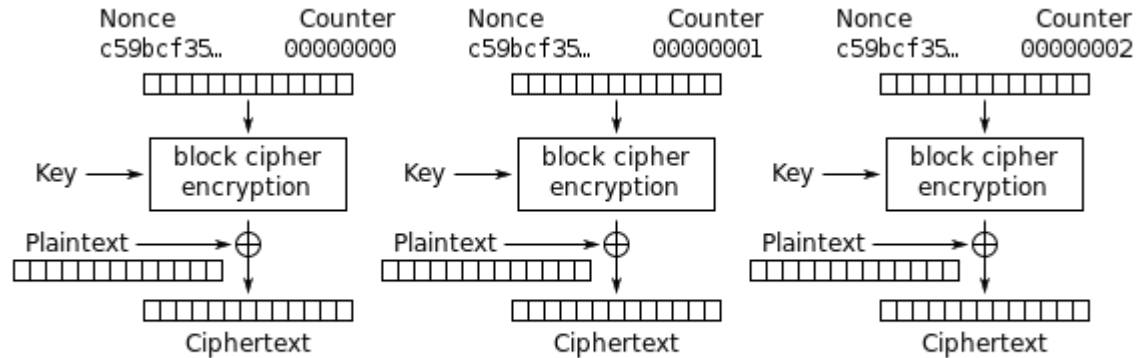


Original image



Encrypted using ECB mode

CTR Mode

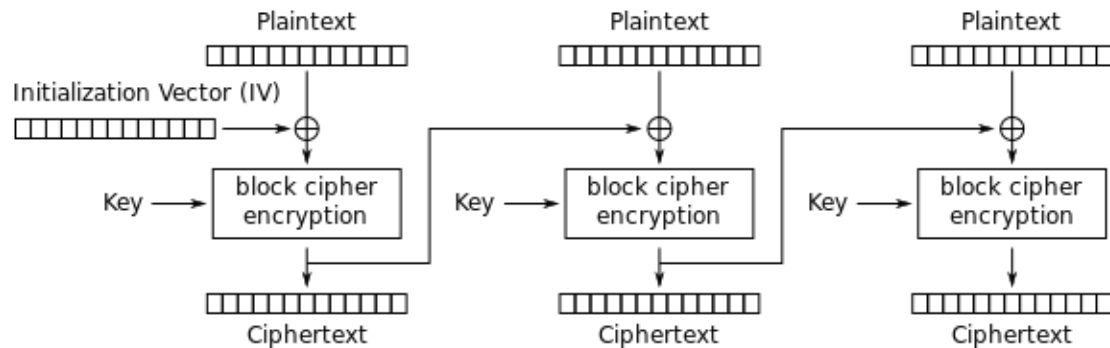


Counter (CTR) mode encryption

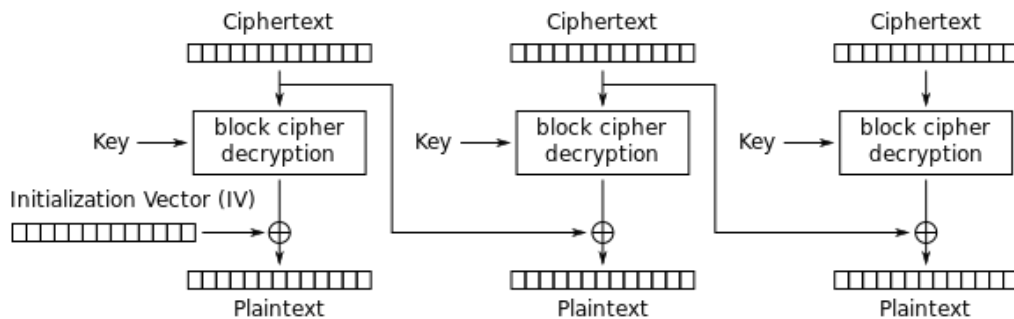
CTR Mode Security Problems

1. 1 p/c pair means game over
 - CTR Mode is like XOR cipher
 - $X = \text{CTR}(\text{key})$
 - $C = P \wedge X$
 - If you have $P_1, C_1, X = C_1 \wedge P_1$
 - If you get $C_2, P_2 = C_2 \wedge X$
2. Bit flipping
 - $P = C \wedge X \Rightarrow P \wedge 1 = (C \wedge 1) \wedge X$

CBC Mode

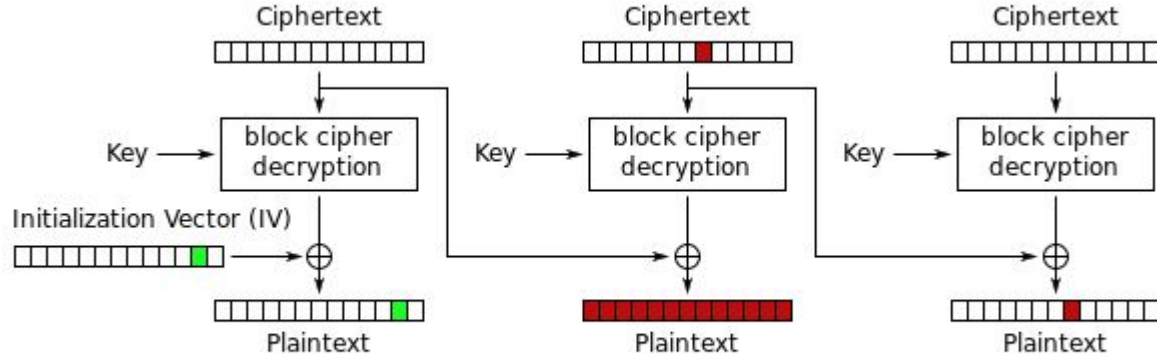


Cipher Block Chaining (CBC) mode encryption



Cipher Block Chaining (CBC) mode decryption

CBC Mode Security Problem - Bitflipping



Cipher Block Chaining (CBC) mode decryption

Padding

- Not all data is of size of multiple of 16 bytes.
- CTR Mode doesn't need padding. (Why?)
- CBC and ECB need padding to be multiple of 16.
- Padding should be such that it is reversible.
- $\text{Pad}(m) \Rightarrow mp$, $\text{Unpad}(mp) \Rightarrow m$, $\text{Size}(mp) = 16k$
- Standard Algo -
 - $\text{Size} = 16k + r$, pad the byte $(16-r)$, $(16-r)$ times

Padding Security Problems - Padding Oracle Attack

- Programs might behave unexpectedly when they receive an invalidly padded data and pass them to unpad.
-

RSA Primer

$N = p * q \Rightarrow$ public key

$\phi(N) = (p - 1) * (q - 1) \Rightarrow$ cannot calculate without p and q individually

ϕ - euler totient function

$a^{\phi(N)} \bmod N = a \bmod N \Rightarrow$ fermat's theorem

$(m^e)^d \bmod N = m \bmod N \Rightarrow e*d = 1 \bmod \phi$

$\text{Enc}(m) = m^e \bmod N$ $\text{Dec}(c) = c^d \bmod N$

Popular RSA Attacks

Observation	Attack
p and q too close (or any linear relation between the two)	$p = \sqrt{N}$ then start increasing until $p \mid N$ (or solve the equation)
$\text{size}(m) * e < \text{size}(N)$	$m = c^{1/e}$ in normal arithmetic (use binary search)
same message m, same e, but e public keys N_i	Use chinese remainder theorem to calculate $c = M^e \pmod{N_1 * N_2 * \dots * N_e}$, then $m = c^{1/e}$ in normal arithmetic
multiple N_s	Try pairwise GCD on all N_i
blinding	
Server decrypts and calculates LSB	LSB Attack

Always try to limit the private key as much as possible!

Shamir Secret Sharing

(n, t) - secret sharing techniques - (Split, Reconstruct)

Split(secret) = $s_1, s_2, s_3, \dots, s_n$

Reconstruct(s_1, s_2, \dots, s_t) = secret (or any t of the n splits)

Shamir - make a random polynomial (p) with constant = secret, of degree (t - 1)

$s_1 = (1, p(1)), s_2 = (2, p(2)) \dots$, we can reconstruct a polynomial with any of degree + 1 = t points

Discrete Logarithm Problem

$$a^x = b \quad \Rightarrow \quad x = \log_a(b)$$

In normal arithmetic (i.e. \mathbb{Z} or \mathbb{N}) x is easy to calculate given a and b - as \log is an increasing function. We can run binary search.

But in discrete settings, things are not easy. (as we saw in RSA)

Diffie Hellman Key Exchange - two people on internet, want to generate a common number only known to these two people.

Party 1 - make a random a , random g , send $(g, g^a \bmod N)$ to second person

Party 2 - make a random b , key = $(g^a \bmod N)^b \bmod N$, send $g^b \bmod N$ to 1

Party 1 - key = $(g^b \bmod N)^a \bmod N$

ECC - Elliptic Curve Group in Discrete Logarithm

- $y^2 = x^3 + ax + b$
- Points on this curve make an element on group, zero point on infinity
- Point Addition ($P + Q$) - connect two points, extend line, where line intersects again, take reflection along x axis, that point is your result
- Point Addition with same point ($P + P$) - tangent instead of line
- Scalar Multiplication (sP) = add s times
- given P and Q, finding s such that $sP = Q$ is hard

Challenges -

<https://github.com/kanav99/csaw19-quals-writeup/tree/master/brillouin-crypto-500>

Name contains hints

sss - shamir secret sharing

lowe - small e

Further Reading

1. CRIME Attack
2. Smart's Attack
3. Learn to use openssl (or maybe during ctf?)